1. 2-D Coin Moving Game (10 points)

Each square of an $n \times n$ board has a pile of zero or more coins on it. A move consists of picking some coins (at least one) on a square and moving them up one square, or to the left one square. The game is over when all the coins are on the upper left square. Give a formula for nimber of a position in this game.

2. 1-D Coin Sliding Game (10 points)

$n$ coins are arranged on a $1 \times m$ grid, but this time coins may not be stacked on top of one another. Adam and Bonnie take turns sliding a single coin any number of spaces to the left (without skipping over other coins, or landing on another coin). When the coins are positioned on the left-most $n$ squares, the last player who moved wins. Give a formula for the nimber of a position in this game.

3. Coin Flipping Game (10 points)

$n$ coins are arranged in a row with either heads or tails showing. Two boys take turns flipping the coins over. During each player’s turn, he must choose a single coin showing heads and flip it to tails. He may then either end his turn immediately, or flip any other coin to the left of the coin he chose and end his turn. When all coins show tails, the player who just moved wins. Give a formula for the nimber of a position in this game.

4. Project Euler Problem 344 (10 points)

This is an optional hard problem. The URL is: https://projecteuler.net/problem=344

Can you solve the problem as stated? You can convert the game to be an impartial normal game by making it illegal to remove the coin to the left of the silver dollar (so the silver dollar can never be taken). In this scenario, can you evaluate the nimber of a position?