Chapter 19

The King and the Consumer

For fools rush in where angels fear to tread.
Alexander Pope, Essay on Criticism.

...because your adversary the devil, as a roaring lion,
walketh about, seeking whom he may devour.
1 Peter 5:8

Figure 1. Chas. Plays Geo.

CHESSGO, KINGGO AND DUKEGO

These games are played on some $i$ by $j$ board. One player, Chas., plays Chess with a lone chess piece which might be a King, or a Knight, or a Duke, or a Ferz, whose moves are shown in Fig. 2. The variants of Chessgo are named for various real and Fairy Chess pieces, Kinggo for a King, etc. Only Kinggo and Dukego will be considered in any detail here.
Chas.'s opponent, Geo., has a number of black (blocking) Go stones and a number of white (wandering) ones. The game starts with the chess piece on a specified square of the otherwise empty board. At each turn the chess piece moves to any legitimate empty square and Geo. then does one of the following:

(a) puts a new Go stone (of either color) on any empty square,
(b) moves a wandering (white) stone already on the board to any other empty square,
(c) passes.

If the chess piece reaches any square on the edge of the board, Chas. wins. If Geo. succeeds in surrounding the chess piece so that it has no legal moves, he wins. A game that continues for ever is declared a draw.

**QUADRAFAGE**

This is the special case, invented by R. Epstein, where there are no wandering stones and enough blocking ones to cover the whole board. The title of this chapter refers to the case of Quadraphage in which the chessperson is the King. In Epstein's language, Geo. is a square-eater (graeco-latin *tesseratore*, latino-greek *quadraphage*). Because Geo. eats a square at every turn, this game ends after at most $ij - 1$ moves on an $i$ by $j$ board. The starting position for the chessperson is conventionally the middle of the board, or as near as possible if $i$ or $j$ is even.

Since having the first move is never a disadvantage, a strategy-copying argument shows that there are only three possible outcomes for a well-played Quadraphage game from a given starting position on a finite board. Either Geo. wins (even if Chas. moves first) or Chas. wins (even if Geo. moves first) or the first player to move wins. A fair position is one in which the first player to move can win.

We'll show that the fair starting positions for the Duke on a quarter-infinite board are all of the squares on the third rank or file, except those that are also on the first or second file or rank. We'll also show that the fair starting positions for the King on this board are all of the squares on the ninth rank or file, except those which are also on a lower file or rank. Finally we'll show that the square board which is fair (from the conventional starting position) for a Duke is the ordinary 8 by 8 chessboard, and we assert that the only fair and square boards for a King are 33 by 33 and 34 by 34. On boards smaller than these, Chas. should win even if Geo. starts first and the reverse should happen on larger boards.
THE ANGEL AND THE SQUARE-EATER

The game of Chessgo is not well understood and it's very difficult to exhibit explicit winning strategies for Chas. even on modest sized boards. For example it seems very likely indeed that the Knight can draw on an infinite board although this seems extremely difficult to prove.

Indeed it's never been shown that there is any generalized chess piece that can draw on the infinite board. This suggests the following problem. An angel (of power 1000) is a chessperson who can fly in one move to any empty square which could be reached by a thousand King moves. Angels, of course, have wings, so it won't matter if some of the intervening squares have been eaten.

![Figure 3. The Angel and the Square-Eater.](image)

We'll say the angel wins by continuing forever (i.e. drawing the game of Quadraphage) against a square-eating devil (who can devour any square of the board, no matter how far away it is from his previous moves). The devil, of course, wins if he can surround the angel with a sulphurous moat, a thousand squares wide, of eaten squares. Can you give an explicit strategy that's guaranteed to win for the angel?

If the devil adopts certain cunning tactics worked out for him by Andreas Blass and John Conway, then infinitely often the angel will find itself decreasing its distance from the centre by arbitrarily large amounts. Although the angel never seems to be in any real danger, its path must also contain arbitrarily convoluted spirals.

STRATEGY AND TACTICS

In both Dukego and Kinggo it's possible to distinguish between strategic moves and tactical ones. In either game Geo. wins, on large enough boards, by first playing a few strategic stones on squares far away from the chess piece. When the chess piece gets closer to the edge of the board, Geo. switches to tactical moves fairly close to him. Whenever the chess piece is driven away from the edge towards the centre of the board, Geo. reverts to strategic moves.
DUKEGO

Dukego is much simpler than Kinggo and so we consider it first. You might like to try playing it yourself before reading this section. The optimal strategies we present here were first discovered by Solomon Golomb. We consider various infinite boards first.

On an infinite half-plane the Duke can win only if he can get to the edge at his first move. In any other situation Geo. can draw by playing directly between the Duke and the edge. In fact Geo. needs only one white (wandering) stone.

On an infinite strip of width $i$ with $i \leq 4$ the Duke, moving first, can win immediately. If $i > 4$ and Geo. moves first he can draw by playing between the Duke and the nearest edge and again needs only one stone, if it's a wandering one.

On an infinite quarter-plane the Duke, moving first, can win if he starts within a three squares wide border. His initial move attacks the edge and Geo. has no choice but to move directly between Duke and edge. The Duke then charges towards the corner. At each move Geo. is forced to play between the Duke and the edge and eventually the Duke wins by reaching one of the two squares next to the corner.

If Geo. moves first against the Duke on the third rank or file of an infinite quarter plane, he can draw using just one blocking stone and one wandering one. He first puts his blocking stone at the strategic position diagonally next to the corner (Fig. 4). This blocks the only square from which the Duke might attack two boundary squares at once. Whenever the Duke moves onto a lower case letter, Geo. puts his wandering stone on the corresponding capital letter.

![Figure 4. Geo. Beats the Duke on a Quarter-Infinite Board.](image)

On an $8 \times j$ board for any value of $j$, the Duke can win if he moves first no matter how many stones Geo. has. The Duke first advances towards the nearest edge, getting onto the third rank or file. If Geo. blocks his advance, the Duke charges along the edge to win in one of the corners. If Geo. does not immediately block his advance, the Duke’s second move places him next to an edge square and Geo. puts a second stone on the board. One of Geo.’s stones must be on the edge square next to the Duke; if the other is to the left of this, the Duke charges to the right, and if to the right, he charges leftward. In either case the Duke eventually wins in the other corner.

On a $7 \times j$ board the Duke can win even if Geo. starts, by attacking whichever half of the board does not contain Geo.’s opening stone.
On the 8 by 8 board Geo. can draw using only three wandering stones (Fig. 5). He always arranges to have his stones on the capital letters corresponding to the small letters covered by the Duke. Since the combinations of small letters on any two contiguous squares never differ by more than one letter, this is always possible. Geo. can draw with just two wandering stones and two blocking ones, by placing the blocking stones on A and C. This works because every square with three letters includes just one of a and c. He can also draw with just one wandering stone and four blocking ones placed at A, B, C, D. It's much harder to find how many blocking stones Geo. needs when he has no wandering ones. Table 1 summarizes the fair starting positions in Dukego.

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<tr>
<th>Size of Board</th>
<th>Starting Position</th>
<th>Least Number of Stones Giving Geo. at least a Draw, Moving First</th>
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<tr>
<td>4 x ∞</td>
<td>centre.</td>
<td>1 wandering.</td>
</tr>
<tr>
<td>quarter-infinite.</td>
<td>3rd rank or file, excluding 1st or 2nd file or rank.</td>
<td>1 wandering, 1 blocking.</td>
</tr>
<tr>
<td>8 x j, j ≥ 8</td>
<td>centre.</td>
<td>3 wandering, or 2 wandering, 2 blocking, or 1 wandering, 4 blocking, or ? blocking.</td>
</tr>
</tbody>
</table>
position of the King. We suppose that the lettered squares are empty; Go stones may occupy any or all of the other squares. In each case Geo. is to move.

If Geo. moves outside the region shown, the King simply advances towards the edge, while if Geo. moves on to a letter $x$ or $x'$ ($x = a, b, ..., g$) the King makes a move which results in a case of Fig. 6(x) or its reflection. If, for example, Geo. puts a stone in the lower right corner (labelled $d'$) of Fig. 6(f), the King moves downwards and achieves a reflexion of Fig. 6(d),

Figure 6. How the King Gets to the Edge.

Figure 7. How the King Wins on an Infinite Strip of Width Eleven.
Now a glance at Fig. 7 and Fig. 6(h) shows that:

the King can win on an infinite strip of width at most 11, even if Geo. goes first.

THE EDGE DEFENCE

Figure 8. How Geo. Guards the Edge Against the King.

Figure 8, which again refers back to Fig. 6, shows that there are only five possible moves (?,?,?,?,?) which give Geo. any chance of stopping the King approaching from the sixth rank of an empty board. Figure 9(k) shows how Geo. can successfully defend the edge with any of these five moves. The King may move from any of the shaded squares. If he remains on such a square Geo. passes. When the King moves on to a letter x or x' (x=j,k,l,...,q) Geo. can move into a case of Fig. 9(x) or its reflection (as he did in Fig. 6). Note that the proof of each of Figs. 9(j) to 9(q) depends on the others, because the King can nip from one of these positions to another in ingenious ways.

Since none of these positions has more than three stones, Geo. can defend the edge with only three wandering stones.

A MEMORYLESS EDGE DEFENCE

Figure 10, which uses the same conventions as Fig. 5, shows another way that Geo. can stop the King approaching from the sixth rank of an empty board using just three wandering stones. Unlike Fig. 9, this is memoryless in the sense that the positions of these stones depend only on the position of the King and not on how he got there.
Figure 9. Three Wandering Stones Ward Off the King.

Figure 10. A Memoryless Kinggo Edge Defence.
In later sections of this chapter Geo. will want to patch together several copies of this defence (Figs. 11 and 12). Figure 11 shows how it may be joined to its left-right mirror image, and Fig. 12 shows how to change its phase by one square. Many other memoryless edge defences can be obtained by joining various combinations of Figs. 10, 11, 12 and their translates and reflections.

Figure 11. Wedding Figure 10 With Its Reflexion.

Figure 12. Getting Figure 10 One Square Out of Step with Itself.

Some results for infinite strips follow immediately from Figs. 9, 10, 11, 12:

On an infinite strip of width at least 12, Geo., moving first, can draw with just 3 wandering stones.

On an infinite strip of width at least 13, he can draw even if the King moves first.
If the King advances towards the edge he will be stopped at Fig. 9(q) and if the King refuses to attack the edge, Geo. can still obtain 3 consecutive stones as in that figure.

(a) At most two stones.
   Not both on A and D,
   Not both on two of A, C, F.

(b) At most one stone.

(c) Stones on two of B, C and D. None elsewhere.

(d) No stones.

(e) No stones.

(f) At most one stone.

(g) At most two stones.

Figure 13. The Edge-Corner Attack.
THE EDGE-CORNER ATTACK

On an infinite strip of width 13, the King can't win, but can force his way to the second rank, as in Fig. 9(q). He may then charge along this rank in either direction, forcing Geo. to accompany him. Even if Geo. has a large supply of stones he can do no more than build up a solid wall along the first rank and on a finite board the edge-charging King will eventually reach a corner.

We now claim that for an adequate defence, Geo. must have at least three strategic stones stationed somewhere between the edge-charging King and the corner. All of these stones must be positioned somewhere in the first five ranks. The proof of this follows from Fig. 13, which lists the appropriate moves for the King against all positions not satisfying these conditions. There are squares with one of the first seven capital letters and infinitely many squares with no letters at all. At Geo.'s move he has at most 3 stones in the figure. The line

"Versus A^0, go to A, position −"

means that if none (superscript 0) of the 3 stones are on A, then the King moves to A and wins at once. The line

"Versus A^1 B^0 C^0 D^<1 E^<1, go to C, position a"

means that if Geo. has one stone on A, none on B or C, and at most one (superscript <1) on each of D and E, the King should move to C and obtain a translate of the position shown in Fig. 13(a).

Since in every case the King counters Geo.'s moves to any of Figs. 13(a) to 13(g) by a move resulting in another of these figures, Geo. can never force the King above the fifth rank or prevent him from continuing the edge-corner attack, although he can keep him moving to and fro among these seven figures.

On the other hand, almost all combinations of three strategic stones along the first rank of the board will suffice for Geo. to stop the edge-corner attack. Figure 14 shows the only exceptions.

![Figure 14. Triplets Which Fail to Stop the Edge-Charging King.](image)

In Fig. 14(a) Geo. has three strategic stones at a1, b1, c1, as well as his tactical stones, h1, i1, ..., defending the edge near the King. The King moves to 1; Geo. is forced to put a stone at 2; the King moves to 3; and so on. The King's move to 9 guarantees him a win by Fig. 6(f) (reflected).
Figure 14(b) uses a similar notation to show how the King wins if Geo.'s strategic stones are at d1, e1, f1.

STRATEGIC AND TACTICAL STONES

Since Geo. can stop an edge corner attack with just three extra stones, and most combinations of three stones suffice, it's convenient to call his three most distant stones in the first five ranks along any edge of the board strategic stones; his other stones are tactical ones. The tactical stones try to stop the King winning along the side and the strategic ones then prevent him from winning the ensuing edge-corner attack.

Let's consider for instance the game on an infinite strip of width 23 (Figure 15). The King starts at 1; Geo. puts a stone at 2; the King moves to 3; Geo. puts a stone at 4; and so on. The crucial position arises when Geo. puts a stone at 16. Where should the King move now? Although various moves look plausible, only one succeeds!

You must distinguish between strategy and tactics if you're to find the right move. The stones 4, 6 and 8 defend the right flank, so 10, 12 and 16 are needed to defend the left one. Since the stone at 16 is required for strategic purposes it is tactically worthless.

So the King pretends that 16 is empty and moves to 17, which would give him a tactical victory via Fig. 6(g)! Any other King move would lose to a defence at α or β.

Of course, since 16 isn't empty, the game won't end on the lower edge, for Geo. can stop the edge attack, but only by using the stone at 16. Eventually the King would have an opportunity to move to 16 if it were vacant. Instead of doing this he embarks on an unstoppable edge-corner attack, running along the second rank towards the left. Geo. can eventually use his stones 10 and 12 to divert the King into various positions of Fig. 13 but can't halt the edge-corner assault.

This sort of argument shows that:

![Box](image)

We believe that this remains true when we remove the constraints on Geo.'s initial moves, since it seems very unlikely that he gains any advantage by putting his stones nearer to the middle. Although such moves seem futile, we haven't managed to exhibit a precise strategy by which the King can refute them.

CORNER TACTICS

Figure 16 shows how Geo. defends the corner against an attack from either edge using three consecutive blocking stones and three wandering ones. The edges can be continued using Fig. 10 to give a strategy for Geo. on a quarter-infinite board.
Figure 15. The King Draws a Typical Game on an Infinite Strip of Width 23.
Although it defends the corner from attack along either edge, it provides only a weak defence against a direct attack towards the corner along the diagonal. It defends against a King on the tenth rank and tenth file of an empty board only when Geo. moves first. Since Geo. must first enter his three strategic stones, if the King moves first he will arrive at the sixth rank and file before Geo. has put any wandering stone on the board, and Fig. 16 now requires Geo. to put wandering stones on both $F$ and $X$. In fact Fig. 17 (which should be used in conjunction with Fig. 6) shows that the King can now win against any strategy for Geo., even if there are stones on all the indicated squares of Fig. 17(a). This figure depends on Figs. 17(b) to 17(e) whose proofs are left to the reader.

Figure 17 shows that Geo.'s only hope of defending the corner against a diagonally attacking King, starting from the tenth rank and file, requires that his first three stones be placed elsewhere. One promising possibility uses squares $a_2$, $a_3$, $a_5$ along one edge, when Geo.'s major problem is to find an appropriate continuation when the King arrives on the sixth rank and file. Figure 18(a) shows that there is only one possibility (indicated by ?). The proof of Fig. 18 depends on Fig. 17 and Fig. 6, but we again leave some of these proofs to the reader.

So there's only one move with which Geo. can successfully defend Fig. 18(a)! His complete strategy appears in Fig. 19 with the edges extended by Fig. 10. With the three blocking stones positioned as shown, he defends the corner against attacks along edges or diagonal.
Figure 17. Three Consecutive Blocking Stones Won't Defend the Corner Against the King on the Sixth Rank and File.

† If Geo. moves here, the King wins via a reflection of Fig. 6h.

Figure 18. With Three Well Placed Stones and the Initial Move, Geo. Defends the Corner Against the Diagonal Attack.
Combining Figs 13 and 19 we have:

the fair starting positions on the quarter-infinite board are those on the ninth rank or file, excluding lesser files or ranks.

Geo., moving first, can defend any such position with just three blocking stones and three wandering ones according to Fig. 19. But if the King moves first, he attacks the nearest edge, ignoring the three further stones between him and the corner as in the sample game of Fig. 15. Since Fig. 13 shows that Geo. needs three strategic stones to defend the corner, the King can either win on the edge or divert to a winning edge-corner assault as in Fig. 13.

DEFENCE ON LARGE SQUARE BOARDS

We've seen that Geo. can defend a corner with three blocking stones and three wandering ones, so he can defend a large enough square board with twelve blocking stones (three in each corner) and three wandering ones. He first puts the twelve blocking stones in their permanent places. If the board is 35 × 35 or bigger, the King begins at least 18 squares from any edge, so he's still at least 6 squares away from the edge after Geo. has placed his 12-strategic stones. So,
Geo., moving first, can win on a square board of size $35 \times 35$ or larger.

**THE $33 \times 33$ BOARD**

![Diagram of $33 \times 33$ board]

**Figure 20.** The Centred King on a $33 \times 33$ Board.
THE KING AND THE CONSUMER

We'll now show you a more intricate defence which allows Geo., moving first, to survive on a $33 \times 33$ board with just 12 (wandering) stones. The details are in Figs. 20 to 26.

THE CENTRED KING

So long as the King stays in the central region of Fig. 20, Geo. puts stones on certain strategic squares, marked with circles on the perimeter of the board. There are 32 of these, 3 near each corner and 5 on each edge. Geo. puts the first four stones one on each edge, and the distribution of his stones after the King has made four or more moves is shown in Fig. 21, a close-up of part of Fig. 20 (the four quarters of the board are congruent). Most of the squares in the central region are divided into nine subsquares, the central one of which is always empty. The other eight subsquares tell Geo. how many stones he should have in each corresponding area. For example, if the King moves to a square marked

```
 3
 1
```

then Geo. moves so that he has three stones on the left edge, one near the bottom left corner and four on the bottom edge. The order in which Geo. puts his stones in the three squares near the corner doesn’t matter, but of the five strategic squares on each edge, it’s the middle one that must be occupied last. A reasonable order is indicated by the numbers 1,2,3,4,5 in the circles in Figs. 20 and 21.

A few squares on the main diagonals of Fig. 21 contain arrows:

```
 1 1 1
 1 2 1
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means

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 1 1 1
 2 2 1
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or

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 1 1 1
 2 2 1
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and

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 1 1 2
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means

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 2 1 1
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or

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 2 1 2
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or

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 3 1 2
```

(but not

```
 0 3 3
```

Geo. can use any of these alternatives as a satisfactory defence.

LEAVING THE CENTRAL REGION

If the King leaves the central region of Fig. 20 via a square marked as in Fig. 22(a), we'll say that he’s cornered in the lower left of the board, and then Geo. will keep him inside the region
shown in Fig. 23 by making tactical moves which prevent the King from reaching a shaded square, so that he can only "re-centre" himself by moving to a square marked as in Fig. 22(e).

Figure 22. Key to Markings in Figures 20, 21, 23, 24, 25 (see text).

Figure 23. The Cornered King.
THE CORNERED KING

If the King moves to squares marked as in Fig. 22(b), (c) or (d) he is correspondingly cornered in the upper left, upper right or lower right of the board. If the King moves to a square labelled as in Fig. 22(f) he is cornered in the lower left or lower right of the board, depending on the direction he came from. If he moves to a label like Fig. 22(g) he is sidelined (see later) and when he moves to one like Fig. 22(h) he is either sidelined or cornered, again depending on which way he came; if diagonally, he's sidelined; if horizontally then he'll be pushed back to the corner whence he came.

THE CORNERED KING

Figure 24, a close-up of Fig. 23, reveals the tactical details that Geo. uses to keep the King cornered with just three wandering stones and nine static ones (semi-stationary, both strategic and tactical). Of course, when the King first becomes cornered by moving to a square marked as in Fig. 22(a), Geo. may not have his nine static stones in the exact places shown in Fig. 24, but he will have three stones between the King and the lower left corner, and three on the bottom edge and three on the left edge. Geo. uses the stones already on the boundary as substitutes for any stones missing from Fig. 24. When the tactics call for placing a stone on a square already occupied, Geo. places a stone on an unoccupied circle in Fig. 24.

Suppose, for example, that the King leaves the central area of Fig. 20 by moving to square k4 (see Fig. 1). He must have come from i5, marked

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<td>3</td>
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<td>5</td>
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so there are already three stones on the left-edge, three as indicated near the lower left corner and five in the squares 2, 4, 5 and those next to Z and A and between them ("3" and "1"). The King is now on a square labelled "s24" so Geo. puts his last stone (the white one in Fig. 1) on S and continues to follow Fig. 24 with the stone on "5" substituting for the missing one between Z and A.

The right arrow in certain squares near the top right of Fig. 24 means that the third genuinely wandering (non-static) stone belongs on a strategic square on the right edge.

THE SIDELINED KING

If the King leaves the central area of Fig. 20 by moving onto a square marked as in Fig. 22(g) he is sidelined as in Fig. 25. Geo. tactically keeps the King off the shaded squares and the King can only re-centre himself by moving onto a square marked as in Fig. 22(e).

The notation in Fig. 26 (a close-up of Fig. 25) is as in Figs. 21 and 24, but we now have some squares

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<td>6j</td>
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Figure 24. Close-up of Figure 23.
These advise Geo. to have one stone on each of the left and right edges (shown in Fig. 25 in their lowest and highest positions) and six on the bottom edge, not only the usual five but one on J or Q as well. [Assume in Fig. 26 that Geo. has 6 static stones on 1, 2, 3, 4, J, Q, and that one of the last two is substituting for 5.]

Figure 25. The Sidelined King.

HOW CHAS. CAN WIN ON A 34 × 34 BOARD

Geo., going first, can survive in Kington with 12 wandering stones on a 33 × 33 board, so he can certainly survive on a 35 × 35 or larger board, even if the King goes first.

However it seems that the King can win on a 34 × 34 board if he moves first. Here's how he does it. His first three moves diagonally attack the nearest corner. He then turns left or right and attacks the adjacent corner in the half of the board where Geo. has at most one stone. After 9 more moves he is at 16, say, and Geo. has been unable to get 9 useful stones on the board. If the corner is adequately defended (with 3 stones), then one flank or the other is weak and a carefully executed edge-corner attack eventually leads to victory.
Unfortunately we haven't been able to formalize these remarks into a strategy for Chas. that's even as explicit as Geo.'s $33 \times 33$ one.

**RECTANGULAR BOARDS**

Geo. can't beat the King on the infinite strip of width 23, even if he moves first and has an unlimited supply of stones. However, if he moves first on a $24 \times n$ board, he can win for sufficiently large $n$. The minimum value of $n$ seems to be about 63. The King is immediately sidelined along whichever long edge he's nearest to. The King can circumvent the pseudocorners ($I$ and $R$ in Fig. 26) of Geo.'s sideline defence, but only by moving back to squares about midway between the two long edges. Geo. can then defend a second pseudocorner between the real corner and the first one. By the time the King reaches the corner, Geo. has prepared defences of both corners along a short edge of the board and a pair of opposite pseudocorners somewhere between the King and the unattacked short edge.

For each value of $i$, $24 \leq i \leq 37$, there appears to be a range of values of $j$ for which the $i$ by $j$ board is a fair battleground for a Quadraphage game against the Chess King. We believe that the $32 \times 33$ board is fair and that Geo., moving first, wins with a strategy similar to that we gave for the $33 \times 33$ board. We leave the problem of determining the dimensions of all fair Quadraphage boards as a challenge for Omar.
EXTRAS

MANY-DIMENSIONAL ANGELS

...can escape from the corresponding hypercubeeaters. This has been proved by Tom Körner who thinks that his proof could conceivably be adapted to the two-dimensional game. Don’t write to us with your solution to the Angel problem unless you’ve taken account of the remarks on p. 609!

GAMES OF ENCIRCLEMENT

The games of this Chapter, and of the next two, are ones of encirclement or escape. There are many games, going a long way back in history, in which this idea is combined with varying kinds of capture. Here are a few examples.

WOLVES-AND-SHEEP

There are several games played on Solitaire-like boards (Chapter 23). In Wolves-and-Sheep (Fig. 27(a)) the shepherd has 20 sheep, which have first move. They move one place forward or sideways only, onto unoccupied places. The two wolves can move similarly but on any of the indicated lines and can capture in these directions by jumping as in checkers (draughts), including multiple captures. A wolf failing to make a possible capture may be removed by the shepherd, so the sheep may be used as decoys. The shepherd wins if he gets nine sheep into the fold (top 9 positions of board).

![Figure 27. Wolves, Sheep and Other Animals.](image-url)
The games shown in Figs. 27(b) and (c) are called Fox and Geese, although we use this name for a different game in the next chapter. They are similar to Wolves-and-Sheep, but there are no diagonal moves. The fox starts in any unoccupied position, and the geese try to crowd the fox into a corner. In Fig. 27(b) the 13 geese can move in any of the four orthogonal directions, but the 17 geese of Fig. 27(c) can’t move backwards; they move like the sheep in Wolves and Sheep.

Hala-taf (the Fox Game), and Freytafl are mentioned in the later Icelandic sagas. As in the Chapter 20 version of Fox and Geese, the more numerous animals win with correct play, but it’s very easy to make mistakes!

**TABLUT**

![Diagram of Tablut](image)

**Figure 28.** The Start of a Game of Tablut.

![Diagram of Muscovite capturing two Swedes](image)

**Figure 29.** A Muscovite Captures Two Swedes.

Linnaeus, on his 1732 visit to Lapland, recorded a game played on a $9 \times 9$ board (Fig. 28) whose centre square, the Konakis or throne, may only be occupied by the Swedish King. He is protected by 8 blond Swedes and confronted by the 16 swarthy Muscovites. All the pieces move like the rook in Chess, any distance orthogonally. Capture of the King is by surrounding him, N, S, E and W by four Muscovites or by three Muscovites with the Konakis as the fourth square. Any other piece is removed by custodian capture, i.e. by placing two opposing pieces to the immediate N and S, or E and W of it. Figure 29 shows a Muscovite capturing two Swedes. A piece may move “into custody” without being captured. The aim of the Swedes is to get their King to the edge of the board.

**SAXON HNEFATAFL**

Only a fragment of a board has been found; it is probable that the game was played using the $19 \times 19$ positions of a modern Go board. See R.C. Bell’s excellent little book for a possible reconstruction from a tenth century English manuscript. The game was evidently like Tablut apart from the size of the board and the number and position of the pieces.

We finish this chapter with two Chess problems which also involve escape or encirclement.
KING AND ROOK VERSUS KING

Most beginning Chess players soon learn how to win this ending, so it’s a surprise to find a couple of non-trivial problems which use just this material, albeit on a quarter-infinite board.

In Fig. 30, can White win? If so, in how few moves? Simon Norton says it’s better to ask, “what is the smallest board (if any) that White can win on if Black is given a win if he walks off the North or East edges of the board?” Can Omar prove that it’s 9 x 11?

Figure 31 shows Leo Moser’s problem: can White win if he’s allowed to make only one move with the Rook? If you find yourself frustrated by this, partition the squares in the first three columns into the four sets
a1,a3,a5,...,c2,c4,c6,...
b1,b3,b5,...
a2,a4,a6,...,c1,c3,c5,...
b2,b4,b6,...

Figure 30. Simon Norton’s Problem.
Figure 31. Leo Moser’s Problem.
REFERENCES AND FURTHER READING


